fg–REGULAR AND fg–NORMAL SPACES

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ABSTRACT :

In this paper, some different types of regular spaces and normal spaces are unified in fuzzy setting. In [9], fuzzy μ-generalized closed (or $f\mu_g$-closed, for short) sets are introduced in a fuzzy topological space (fts, for short) in the sense of Chang [11]. In [6], fuzzy generalized μ-closed sets have been introduced and studied. In this paper we firstly have shown that fuzzy generalized μ-closed sets and $f\mu_g$-closed sets are independent notions. Finally, fuzzy generalized regular (fg-regular, for short) space and fuzzy generalized normal (fg-normal, for short) space have been introduced and studied.

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KEYWORDS: Fuzzy μ-open set, fuzzy μ-generalized closed set, fuzzy generalized μ-closed set, fg-regular space, fg-normal space.

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INTRODUCTION

In [14], fuzzy regular space has been introduced and studied. Afterwards, many researchers have engaged themselves for introducing different types of regular spaces in fuzzy setting by replacing fuzzy open set by fuzzy semiopen set [1], fuzzy δ-preopen set [3], fuzzy α-open set [10], fuzzy β-open set [2] respectively and as a result fuzzy s-regular, fuzzy δ-preregular [4], fuzzy α-regular [7], fuzzy β-regular [8] spaces have been introduced. Again, in [13], Hutton has introduced and studied fuzzy normal space. In the same way one can introduce fuzzy p-normal, fuzzy s-normal, fuzzy α-normal, fuzzy β-normal, fuzzy δ-normal, fuzzy δ-prenormal, fuzzy θ-normal spaces by replacing fuzzy open set by fuzzy preopen [15], fuzzy semiopen, fuzzy α-open, fuzzy β-open, fuzzy δ-open, fuzzy δ-preopen and fuzzy θ-open sets respectively.

Owing to the fact that the corresponding definitions have many features in common, it is quite natural to conjecture that they can be unified in a suitable way. This paper plays an important role in this regard.

PRELIMINARIES

Let us now recall some notions for ready references.

Let X be a nonempty set and I^X denote the set of all fuzzy sets [18] in X. We call a class $\mu \in I^X$, a fuzzy generalized topology (FGT, for short) [6] if $0_X \in \mu$ and μ is closed under arbitrary union. Then (X, μ) is called a fuzzy generalized topological space (FGTS, for short). The support of a fuzzy set A in X will be denoted by $suppA$ [17] and is defined by $suppA = \{x \in X : A(x) \neq 0\}$. A fuzzy point [17] with the singleton support $x \in X$ and the value α $(0 < \alpha \leq 1)$ at x will be denoted by x_α. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively. The complement [18] of a fuzzy set A in X will be denoted by $1_X \setminus A$and is defined by $(1_X \setminus A)(x) = 1 - A(x)$, for all $x \in X$. For any two fuzzy sets A and B in X, we write $A \leq B$ if and only if $A(x) \leq B(x)$, for each $x \in X$, and AqB means A is quasi-coincident (q-coincident, for short) with B if $A(x) + B(x) > 1$, for some $x \in X$; the negation of these two statements are denoted by $A \nleq B$ and $A\bar{q}B$ respectively. clA and $intA$ of a fuzzy set A in X respectively stand for the fuzzy closure and fuzzy interior of A in X[18].

A fuzzy set A in X is called fuzzy regular open [1] if $A = cl\, intA$.A fuzzy set A in X is said to be fuzzy semiopen [1] if there exists a fuzzy open set U in X such that $U \leq A \leq clU$, or equivalently, if $A \leq cl\, intA$. The fuzzy θ-closure [16] (resp., fuzzy δ-closure [12]) denoted by θcl (resp., δcl) of a fuzzy set A in an fts (X,τ) is the union of all those fuzzy points x_α such that $clUqA$ whenever $x_\alpha qU \in \tau$ (resp., UqA whenever $x_\alpha qU$ where U is fuzzy regular open set in X). A fuzzy set A is called fuzzy θ-closed [16] (resp., fuzzy δ-closed [12]) if $A = \theta clA$ (resp., $A = \delta clA$) and the complement of a fuzzy θ-closed (resp., fuzzy δ-closed) set is known as a fuzzy θ-open [16] (resp., fuzzy δ-open [12]) set. A fuzzy set A in an fts (X,τ) is called fuzzy preopen [15] (resp., fuzzy δ-preopen [3], fuzzy α-open [10], fuzzy β-open [2]) if $A \leq int\, clA$ (resp., $A \leq int\, \delta clA$, $A \leq int\, cl\, intA$, $A \leq cl\, int\, clA$). We note that for an fts (X,τ), the collection of all fuzzy open (resp., fuzzy preopen, fuzzy semiopen, fuzzy δ-open, fuzzy δ-preopen, fuzzy α-open, fuzzy β-open, fuzzy θ-open) set is denoted by τ (resp., $FPO(X)$, $FSO(X)$, $F\delta O(X)$, $F\delta PO(X)$, $F\alpha O(X), F\beta O(X)$, $F\theta O(X)$). Each of these collections is an FGT. For an FGTS (X,μ), the elements of μ are called fuzzy μ-open sets and the complements of fuzzy μ-open sets are called fuzzy μ-closed sets. For $A \in I^X$, we denote by $c_\mu(A)$, the infimum of all fuzzy μ-closed sets B with $A \leq B$, i.e., $c_\mu(A) = inf\{\mathrm{B} : \mathrm{A} \leq \mathrm{B},\ \mathrm{B} \in \mu^c\}$; and by $i_\mu(A)$, the supremum of all fuzzy μ-open sets B with $B \leq A$, i.e., $i_\mu(A) = sup\{B : B \leq A, B \in \mu\}$. In an fts (X,τ), if one takes τ as the FGT, then c_μ becomes the usual fuzzy closure operator. Similarly, c_μ becomes fuzzy pcl (resp., fuzzy scl, fuzzy δcl, fuzzy δpcl, fuzzy αcl, fuzzy βcl, fuzzy θcl) if μ stands for $FPO(X)$ (resp., $FSO(X)$, $F\delta O(X)$, $F\delta PO(X)$, $F\alpha O(X), F\beta O(X)$, $F\theta O(X)$).

It is clear that i_μ and c_μ are idempotent and monotonic where $\gamma : I^X \to I^X$ is said to be idempotent if for any two fuzzy sets A and B in X, $A \leq B \implies \gamma(\gamma(A)) = \gamma(A)$ and monotonic if $\gamma(A) \leq \gamma(B)$.

§1. fg_μ-CLOSED SET AND $f\mu_g$-CLOSED SET

We recall first some definitions, theorem and result from [5], [6] and [9].

DEFINITION 1.1 [5]. Let (X,τ) be an fts. Then $A \in I^X$ is said to be fuzzy generalized closed (fg-closed, for short) if $clA \leq U$ whenever $A \leq U \in \tau$. The complement of fg-closed set is called fg-open set.

DEFINITION 1.2 [6]. Let (X,μ) be an FGTS. Then $A \in I^X$ is called a fuzzy generalized μ-closed set (fg_μ-closed, for short) if $c_\mu(A) \leq U$ whenever $A \leq U \in \mu$. The complement of an fg_μ-closed set is called a fuzzy generalized μ-open set (fg_μ-open set, for short).

DEFINITION 1.3 [9]. Let (X,τ)be an fts and μ be an FGT on X. Then $A \in I^X$ is called a fuzzy μ-generalized closed (or simply $f\mu_g$-closed) set if $c_\mu(A) \leq U$ whenever $A \leq U \in \tau$. The complement of an $f\mu_g$-closed set is called a fuzzy μ-generalized open (or simply $f\mu_g$-open) set.

The following two examples show that fg_μ-closedness and $f\mu_g$-closedness are two independent notions.

EXAMPLE 1.4. Let $X = \{a,b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.6$. Then (X,τ) is an fts. If $\mu = \{0_X, 1_X, B\}$ where $B(a) = 0.5, B(b) = 0.4$, then $\mu$is an FGT on X. We claim that B is $f\mu_g$-closed but not fg_μ-closed. Infact, $B \leq A \in \tau$ and $c_\mu(B) = 1_X \backslash B = A$. Therefore, B is $f\mu_g$-closed. But $c_\mu(B) = 1_X \backslash B \nleq B$ and so B is not fg_μ-closed.

EXAMPLE 1.5. Let $X = \{a,b\}$, $\tau = \{0_X, 1_X, A\}$ where $A(a) = 0.5, A(b) = 0.7$. Then (X,τ) is an fts. If $\mu = \{0_X, 1_X, B\}$ where $B(a) = 0.5, B(b) = 0.4$, then μ is an FGT on X. Now $A \leq A \in$

τ and $c_\mu(A) = 1_X \nleq A$ and so A is not $f\mu_g$-closed. Now 1_X is the only fuzzy μ-open set such that $A \leq 1_X$ and $c_\mu(A) = 1_X \leq 1_X$ and so A is fg_μ-closed.

THEOREM 1.6 [9]. Let (X, τ) be an fts and μ be an FGT on X. Then A is $f\mu_g$-open iff $F \leq i_\mu(A)$ whenever $F \leq A$ and F is fuzzy closed in X.

RESULT 1.7 [9]. Let (X, τ) be an fts and $A \in \tau^c$ and B be any fg-open set such that $A \leq B$. Then $A \leq intB$.

§ 2. PROPERTIES OF fg-REGULAR AND fg-NORMAL SPACES

DEFINITION 2.1. Let (X, τ) be an fts and μ be an FGT on X. Then (X, τ) is said to be fg-regular if for each fuzzy point x_α and each fuzzy closed set F of X with $x_\alpha \notin F$, there exist two fuzzy μ-open sets U and V such that $x_\alpha q U$, $F \leq V$ and $U \bar{q} V$.

THEOREM 2.2. Let μ be an FGT on an fts (X, τ). Then the following statements are equivalent:

(a) X is fg-regular.

(b) For each fuzzy point x_α in X and each fuzzy open set U in X with $x_\alpha q U$, there exists $V \in \mu$ such that $x_\alpha q V \leq c_\mu(V) \leq U$.

(c) For each fuzzy closed set F of X, $F = \wedge \{ c_\mu(V) : F \leq V \in \mu\}$.

(d) For any fuzzy set A and any $U \in \tau$ with AqU, there exists $V \in \mu$ such that $AqV \leq c_\mu(V) \leq U$.

(e) For any fuzzy set $A\ (\neq 0_X)$ and each $F \in \tau^c$ with $A \nleq F$, there exist $V, W \in \mu$ such that AqV, $F \leq W$ and $V \bar{q} W$.

(f) For any fuzzy point x_α and any fuzzy closed set F with $x_\alpha \notin F$, there exist $U \in \mu$ and an $f\mu_g$-open set V such that $x_\alpha qU, F \leq V$ and $U \bar{q} V$.

(g) For any fuzzy set A and any fuzzy closed set F with $A\bar{q}F$, there exist $U \in \mu$ and an $f\mu_g$-open set V such that $AqU, F \leq V$ and $U \bar{q} V$.

PROOF. (a) ⇒ (b) : Let x_α be a fuzzy point in X and $U \in \tau$ be such that $x_\alpha qU$. Then $U(x) + \alpha > 1 \Longrightarrow x_\alpha \notin 1_X \backslash U \in \tau^c$. By (a), there exist fuzzy μ-open sets V and W such that $x_\alpha qV, 1_X \backslash U \leq W$ and $V \bar{q} W$. Then $V(x) + W(x) \leq 1$, for all $x \in X \Longrightarrow V \leq 1_X \backslash W \in \mu^c$. Therefore, $V \leq c_\mu(V) \leq c_\mu(1_X \backslash W) = 1_X \backslash W \leq U$. Then $x_\alpha qV \leq c_\mu(V) \leq U$.

(b) ⇒ (c) : Let F be a fuzzy closed set in X. Then $1_X \backslash F \in \tau$. Let $x_\alpha \notin F$.Then $F(x) < \alpha \Longrightarrow x_\alpha q(1_X \backslash F)$. Then by (b), there exists $V \in \mu$ such that $x_\alpha qV \leq c_\mu(V) \leq 1_X \backslash F$. Then $F \leq 1_X \backslash c_\mu(V) = U$ (say) $\in \mu$ and $U\bar{q}V$. So there exists $V \in \mu$ be such that $x_\alpha qV$ and $V\bar{q}U$. Therefore, $x_\alpha \notin c_\mu(U) = \wedge \{c_\mu(U): F \leq U \in \mu\} \leq F$.

Again, $F \leq \wedge \{c_\mu(U): F \leq U \in \mu\}$ is obvious. Hence$F = \wedge \{ c_\mu(U): F \leq U \in \mu\}$.

(c) ⇒ (d) : Let A be any fuzzy set in X and $U \in \tau$ with AqU. Then there exists $x \in X$ such that $A(x) + U(x) > 1$. Let $A(x) = \alpha$. Then $x_\alpha \in A$ and $x_\alpha \notin 1_X \backslash U \in \tau^c$. Then by (c), there exists $W \in \mu$ such that $1_X \backslash U \leq W$ … (1) and $x_\alpha \notin c_\mu(W)$. Therefore, $c_\mu(W)(x) < \alpha \Longrightarrow 1 - c_\mu(W)(x) > 1 - \alpha \Longrightarrow x_\alpha \, q \, i_\mu(1_X \backslash W)$ where $i_\mu(1_X \backslash W) \in \mu$. Take $i_\mu(1_X \backslash W) = V$. Then $V \in \mu$ be such that $x_\alpha qV$ and so $V(x) + \alpha > 1 \Longrightarrow V(x) + A(x) > 1 \Longrightarrow AqV$. Now $V = i_\mu(1_X \backslash W) \leq 1_X \backslash W$ and so $c_\mu(V) \leq c_\mu(1_X \backslash W) = 1_X \backslash W$ ($\in \mu^c$) $\leq U$ (by (1)).

(d) ⇒ (e) : Let A ($\neq 0_X$) be any fuzzy set in X and $F \in \tau^c$ with $A \nleq F$. Then there exists $x \in X$ such that$A(x) > F(x)$. Therefore, $1 - A(x) < 1 - F(x) \Longrightarrow Aq(1_X \backslash F) \in \tau$. Then by (d), there exists $V \in \mu$ such that $AqV \leq c_\mu(V) \leq 1_X \backslash F$. Now $c_\mu(V) \leq 1_X \backslash F \Longrightarrow F \leq$

$1_X \backslash c_\mu(V) = i_\mu(1_X \backslash V) \in \mu$. Let $i_\mu(1_X \backslash V) = W$. Then $F \leq W \in \mu$. Again $V \leq c_\mu(V) \Longrightarrow 1_X \backslash c_\mu(V) \leq 1_X \backslash V \Longrightarrow W\bar{q}V$.

(e) ⇒ (a) : Let x_α be any fuzzy point in X and $F \in \tau^c$ with $x_\alpha \notin F$. Then $F(x) < \alpha$. Then x_α is a fuzzy set such that $x_\alpha \nleq F$. Then by (e), there exist $V, W \in \mu$ such that $x_\alpha qV, F \leq W$ and $V\bar{q}W$. Hence (a) follows.

(d) ⇒ (f) : The proof follows from the fact that that every fuzzy μ-closed set is $f\mu_g$-closed set.

(f) ⇒ (g) : Let $A \in I^X$ and $F \in \tau^c$ be such that $A \nleq F$. Then there exists $x \in X$ such that $A(x) > F(x)$. Let $A(x) = \alpha$. Then $x_\alpha \in A$ and $x_\alpha \notin F$. By (f), there exist $U \in \mu$ and an $f\mu_g$-open set V such that $x_\alpha qU, F \leq V$ and $U\bar{q}V$. Then $U(x) + \alpha > 1 \Longrightarrow U(x) + A(x) > 1 \Rightarrow AqU$.

(g) ⇒ (a) : Let x_α be a fuzzy point in X and $F \in \tau^c$ with $x_\alpha \notin F$. Then $F(x) < \alpha$. Then x_α is a fuzzy set in X such that $x_\alpha \nleq F$. Then by (g), there exist $U \in \mu$ and an $f\mu_g$-open set V such that $x_\alpha qU$, $F \leq V$ and $U\bar{q}V$. Now put $i_\mu(V) = W$. Then $F \leq W$ (by Theorem 1.6) and hence $W\bar{q}U$. Therefore, X is fg-regular.

DEFINITION 2.3. Let μ be an FGT on an fts (X, τ). Then (X, τ) is said to be fuzzy generalized normal (fg-normal, for short) if for any two fuzzy closed sets A and B in X with $A\bar{q}B$, there exist two fuzzy μ-open sets U and V such that $A \leq U, B \leq V$ and $U\bar{q}V$.

THEOREM 2.4. Let μ be an FGT on an fts (X, τ). Then the following statements are equivalent:

(a) X is fg-normal.

(b) For any pair of fuzzy closed sets A, B of X with $A\bar{q}B$, there exist $f\mu_g$-open sets U and V of X such that $A \leq U, B \leq V$ and $U\bar{q}V$.

(c) For each fuzzy closed set A and each fuzzy open set B in X with $A \leq B$, there exists an $f\mu_g$-open set U such that $A \leq U \leq c_\mu(U) \leq B$.

(d) For each fuzzy closed set A and each fg-open set B in X with $A \leq B$, there exists a fuzzy μ-open set U such that $A \leq U \leq c_\mu(U) \leq intB$.

(e) For each fuzzy closed set A and each fg-open set B in X with $A \leq B$, there exists an $f\mu_g$-open set G such that $A \leq G \leq c_\mu(G) \leq intB$.

(f) For each fg-closed set A in X and each $B \in \tau$ with $A \leq B$, there exists a fuzzy μ-open set U such that $clA \leq U \leq c_\mu(U) \leq B$.

(g) For each fg-closed set A in X and each $B \in \tau$ with $A \leq B$, there exists an $f\mu_g$-open set G such that $clA \leq G \leq c_\mu(G) \leq B$.

PROOF. (a) ⇒ (b) : Let A and B be two fuzzy closed sets in X with $A\bar{q}B$. By (a), there exist two fuzzy μ-open sets U and V such that $A \leq U, B \leq V$ and $U\,\bar{q}V$. Then the rest follows from the fact that every fuzzy μ-closed set is $f\mu_g$-closed set.

(b) ⇒ (c) : Let $A \in \tau^c$ and $B \in \tau$ with $A \leq B$. Then $1_X\backslash B \leq 1_X\backslash A \Longrightarrow A\bar{q}(1_X\backslash B) \in \tau^c$. Then by (b), there exist $f\mu_g$-open sets U and V of X such that $A \leq U, 1_X\backslash B \leq V$ and $U\bar{q}V \Longrightarrow U \leq 1_X\backslash V$. Now $1_X\backslash B \leq V$ and V is $f\mu_g$-open, $1_X\backslash B \in \tau^c$. Then by Theorem 1.6, $1_X\backslash B \leq i_\mu(V) \Longrightarrow 1_X\backslash i_\mu(V) \leq B \Longrightarrow c_\mu(1_X\backslash V) \leq B$. Therefore, $A \leq U \leq c_\mu(U) \leq c_\mu(1_X\backslash V) \leq B$.

(c) ⇒ (a) : Let $A, B \in \tau^c$ be such that $A\bar{q}B$. Then $A \leq 1_X\backslash B \in \tau$. By (c), there exists an $f\mu_g$-open set U such that $A \leq U \leq c_\mu(U) \leq 1_X\backslash B$. Now $c_\mu(U) \leq 1_X\backslash B \Longrightarrow B \leq 1_X\backslash c_\mu(U) = i_\mu(1_X\backslash U) \in \mu$. By Theorem 1.6, $A \leq i_\mu(U)$ and $i_\mu(U)\bar{q}(1_X\backslash c_\mu(U))$.

(d) ⇒ (e) ⇒ (b) : Obvious (as fuzzy closed sets are fg-closed).

(f) ⇒ (g) ⇒ (c) : Obvious (as fuzzy closed sets are fg-closed).

(c) ⇒ (e) : Let $A \in \tau^c$ and B be an fg-open set in X with $A \leq B$. By Result 1.7, $A \leq intB$. By (c), there exists an $f\mu_g$-open set U such that $A \leq U \leq c_\mu(U) \leq intB$.

(e) ⇒ (f) : Let A be fg-closed set and $B \in \tau$ with $A \leq B$. Then $clA \leq B$ where B is fg-open (as every fuzzy open set is fg-open set). By (e), there exists an $f\mu_g$-open set G such that $clA \leq G \leq c_\mu(G) \leq intB = B$. Since G is $f\mu_g$-open and $clA \leq G$, by Theorem 1.6, $clA \leq i_\mu(G) \in \mu$. Put $i_\mu(G) = U$. Then $U \in \mu$ and $clA \leq U \leq c_\mu(U) = c_\mu\left(i_\mu(G)\right) \leq c_\mu(G) \leq B$.

(f) ⇒ (d) : Let $A \in \tau^c$ and B be an fg-open set in X with $A \leq B$. By Result 1.7, $clA = A \leq intB$ where A is fg-closed (as it is fuzzy closed). By (f), there exists $U \in \mu$ such that $clA = A \leq U \leq c_\mu(U) \leq intB$.

§ 3. APPLICATIONS

Let us recall some definitions for ready references.

DEFINITION 3.1. An fts (X, τ) is said to be fuzzy regular [14] (resp., fuzzy δ-preregular [4], fuzzy s-regular, fuzzy α-regular [7], fuzzy β-regular [8]) if for each fuzzy point x_α and each fuzzy closed (resp., fuzzy δ-preclosed, fuzzy semiclosed, fuzzy α-closed, fuzzy β-closed) set F such that $x_\alpha \notin F$, there exist fuzzy open (resp., fuzzy δ-preopen, fuzzy semiopen, fuzzy α-open, fuzzy β-open) sets U and V such that $x_\alpha q U, F \leq V$ and $U \bar{q} V$.

REMARK 3.2. It is clear from Definition 2.1 and Definition 3.1 that fg-regular space is the unified version of fuzzy regular, fuzzy δ-preregular, fuzzy s-regular, fuzzy α-regular, fuzzy β-regular spaces.

DEFINITION 3.3. An fts (X, τ) is said to be fuzzy normal [13] (resp., fuzzy p-normal, fuzzy s-normal, fuzzy α-normal, fuzzy β-normal, fuzzy δ-normal, fuzzy δ-prenormal, fuzzy θ-normal) space if for any two fuzzy closed (resp., fuzzy preclosed, fuzzy semiclosed, fuzzy α-closed, fuzzy β-closed, fuzzy δ-closed, fuzzy δ-preclosed, fuzzy θ-closed) sets A and B with $A\bar{q}B$, there exist two fuzzy open (resp., fuzzy preopen, fuzzy semiopen, fuzzy α-open, fuzzy β-open, fuzzy δ-open, fuzzy δ-preopen, fuzzy θ-open) sets U and V such that $A \leq U, B \leq V$ and $U\bar{q}V$.

REMARK 3.4. It is clear from Definition 2.3 and Definition 3.3 that if we take $\mu = \tau$ (resp., $FPO(X)$, $FSO(X)$, $F\delta O(X)$, $F\delta PO(X)$, $F\alpha O(X), F\beta O(X)$, $F\theta O(X)$), we get fuzzy normal (resp., fuzzy p-normal, fuzzy s-normal, , fuzzy δ-normal, fuzzy δ-prenormal, fuzzy α-normal, fuzzy β-normal, fuzzy θ-normal) space.

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